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Edge effect on the power law distribution of granular avalanches

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Many punctuated phenomena in nature are claimed [e.g., by the theory of self-organized criticality (SOC)] to be power-law distributed. In our experiments on a three-dimensional pile of long-grained rice, we find that by only changing the boundary condition of the system, we switch from such power-law-distributed avalanche sizes to quasiperiodic system-spanning avalanches. Conversely, by removing ledges the incidence of system-spanning avalanches is significantly reduced. This may offer a perspective on new avalanche prevention schemes. In addition, our findings may help to explain why the archetype of SOC, the sandpile, was found to have power-law-distributed avalanches in some experiments, while in other experiments quasiperiodic system-spanning avalanches were found.

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Power laws are claimed [1] to describe the size distribution of many events in nature such as solar flares [2], stock market crashes [3], earthquakes [4], and forest fires [5]. The paradigm of such behavior is the sandpile with its avalanches. While the numerical sandpile models [6] indeed seem [7] to show power-law behavior, this is highly debated for real sandpiles. The first sandpile experiments carried out in search of a power-law behavior were performed by Jaeger, Liu, and Nagel [8,9]. Here we refer only to their studies of sandpiles in a half open box. The size of the over-the-rim avalanches was measured using a pair of capacitor plates placed below the edge of the pile. Contrary to theoretical sandpiles, they found a peaked distribution of avalanche sizes due to the system spanning avalanches that dominate the distribution. By analyzing the waiting times between these large avalanches, they observed a narrow Gaussian distribution, showing that these avalanches are quasiperiodic (see Fig. 2 in Ref. [8]). Later Held *et al.* [10] carried out experiments on conical sandpiles with varying diameters. The sizes of the over-the-rim avalanches were calculated from the fluctuations in the mass of the pile. The same quasiperiodic behavior was found for large piles, while the small piles showed a power-law distribution of avalanche sizes. The occurrence of power-law statistics in these small systems is claimed to be a finite-size effect [9,11]. Rosendahl *et al.* [12] conducted the same type of experiments on conical piles. They find even for large system sizes apparently [13] power-law distributed avalanches, in addition to the quasiperiodic system spanning avalanches. However, also in these experiments, the internal avalanches—i.e., the ones that do not reach the edge of the system—were not taken into account. Another type of experiment was performed by Bretz *et al.* [14], who studied the surface of a sandpile placed on a slowly tilting tray. The size of the avalanches was determined from the intensity difference between two consecutive images taken with a charge-coupled-device (CCD) camera, allowing the detection of internal avalanches. They also observed large avalanches that occur in regular intervals next to smaller avalanches, which are power-law distributed. However the distribution spans less than one order of magnitude. All the above sandpile experiments have one common feature: they exhibit large, quasiperiodic avalanches that span the whole surface of the pile (for a comprehensive review of these sandpile experiments see Ref. [13]). As the sandpile experiments failed to exhibit a clear power-law behavior [13], Frette *et al.* [15] carried out experiments on a rice pile confined between two glass plates that are less than a grain length apart. Here also the size of the internal avalanches was measured by following with a CCD camera, from one side, the fluctuations of the surface of the pile. They have observed that the distribution of avalanche sizes depends on the shape of the grains of the pile, finding a power law for long-grained rice and stretched exponential for spherical grains. These rice pile experiments show that next to the importance of detecting the internal avalanches (which is also indicated by numerical simulations—e.g., by Ref. [16]), kinetic effects seem to play an important role in the behavior of a granular pile. Costello *et al.* [17] found in conical piles quasiperiodic avalanches for small cohesive grains, while for large noncohesive beads a power-law distribution emerged. Clearly, for real sandpiles, there are a number of parameters that influence the avalanche size distribution.

In this Rapid Communication we present our experiments on a three-dimensional pile of long-grained rice. We find that there is an additional factor for obtaining SOC behavior in a granular pile: the boundary condition of the system. If the foot of the pile rests on a flat surface, we observe power-law-distributed avalanches over almost four orders of magnitude. However, if the foot of the pile is at the edge of the surface on which the pile rests (such that grains can fall off the ledge), we observe quasiperiodic system-spanning avalanches, in addition to the power-law-distributed small- and medium-sized avalanches. Note that this boundary condition is similar to the one used in some of the sandpile experiments.

Our experiments were carried out on a three-dimensional pile of rice (Silvo Surinaamse rijst), with a typical grain size of $\sim 2 \times 2 \times 7$ mm^3, similar to rice A of Ref. [15]. The pile is contained in a box with three closed sidewalls and a floor area of 1×1 m^2. The fourth side is open, and there rice can leave the box unimpeded. At the opposite side, grains are added slowly at the top of the pile, uniformly across its width (the rate is 30 grains per second per meter width, corresponding to "slow driving" in the SOC sense). The surface $z(x,y)$ of the pile is reconstructed by means of monocular ste-

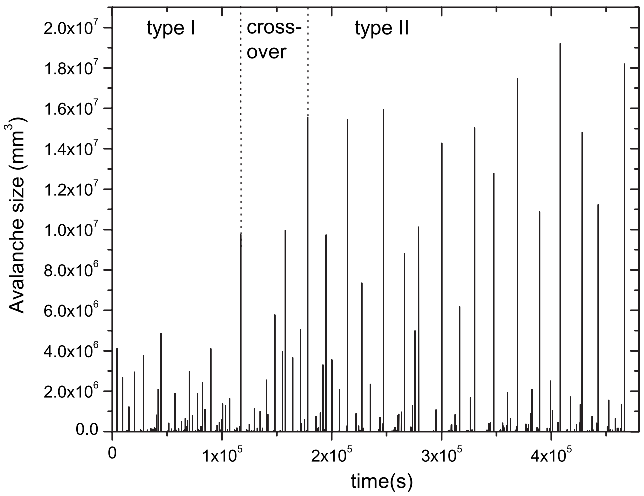

FIG. 1. The evolution of avalanche sizes during one experiment. The dotted lines separate the experiment into three segments. In the first segment the pile is in the type-I boundary condition, followed by a crossover period when part of the foot of the pile reaches the edge of the box and part of it is away from it. In the last segment, the foot is touching the edge along its whole width: the type-II boundary condition. Note the quasiperiodic very large (system spanning) avalanches for the type-II boundary condition.

reoscopy. A set of colored lines is projected on the pile with the direction of projection along the normal to the pile surface, while a high-resolution (2560×1920 pixels) CCD camera takes images of the pile every 17 s at a 33° angle with respect to the direction of projection. The time interval between two images is much shorter than the interval between avalanches. The size of an avalanche is defined as the volume of rice displaced between two consecutive time steps. The experiments start with the "type-I" boundary condition, where the foot of the pile is well away from the open side of the box; data collection starts after a short equilibration time. When the foot of the pile partly reaches the edge (the foot is not straight due to the roughening of the surface by the avalanches) we have a rather complex boundary condition (called "crossover" below), which evolves by definition to the "type-II" boundary condition as soon as the foot of the pile touches the edge over its whole width. From that moment onwards the foot coincides with the edge and is straight.

In Fig. 1, we show the evolution of the avalanche sizes for the longest experiment. First we have the type-I boundary condition with avalanches of all sizes, followed by a crossover period. Due to the complex nature of the avalanches in the crossover period, they were not taken into consideration for further analysis. For the type-II boundary condition, there are larger avalanches, which span the whole system and occur quasiperiodically. This is the same boundary condition that was used in the half-open-box sandpile experiment from, e.g., Ref. [8], where quasiperiodic behavior was found. The noncumulative size distribution of type-I avalanches from 11 experiments (with a total number of 1100 avalanches) is a power law $P(s) \sim s^{-\tau}$ over almost four orders of magnitude [see Fig. 2(a)] with an exponent $\tau = 1.12(2)$. The deviation from power-law behavior in the small-avalanche regime is

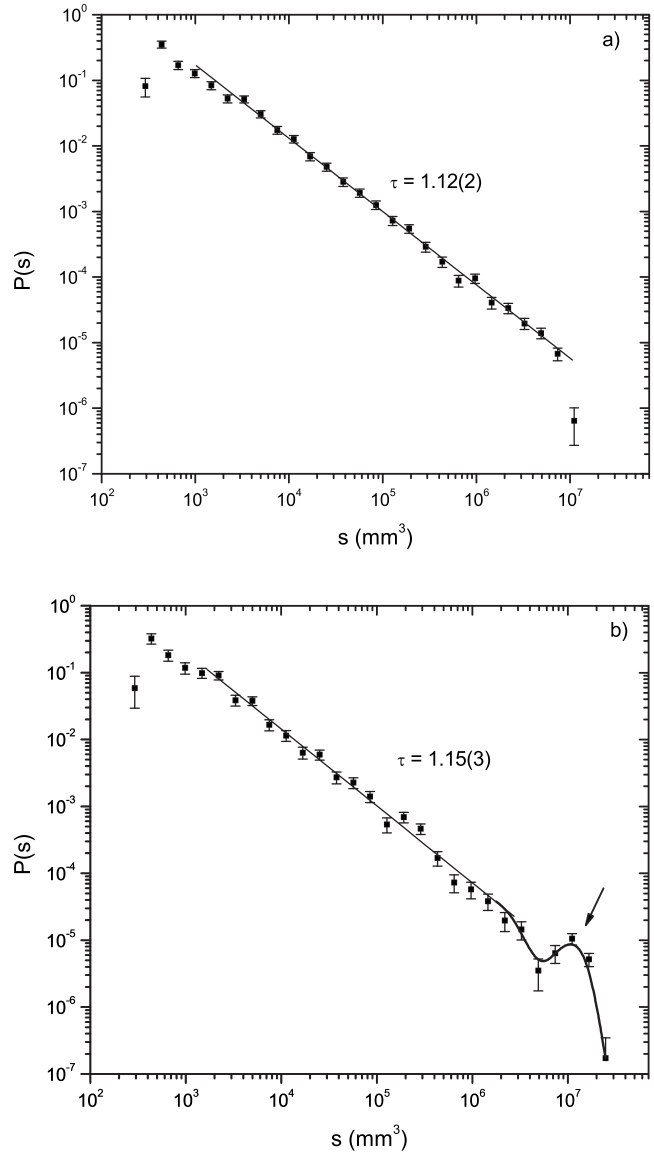

FIG. 2. The noncumulative size distribution of avalanches with (a) type-I and (b) type-II boundary conditions. The straight lines are best fits to the data, corresponding to the power-law behavior $P(s) \sim s^{-\tau}$, with τ indicated in the figure. Note the small, but highly significant deviation from the power law at very large sizes, indicated by the arrow in panel (b). Data shown are for a total of (a) 1100 and (b) 629 avalanches. The error bars are obtained using Poisson statistics; the error in the exponent is the standard deviation from the least-squares fit. The thick black curve is a guide to the eye.

due to the experimental difficulty of identifying small avalanches [18]. For the type-II boundary condition, the size distribution of avalanches from seven experiments (the other four experiments were stopped during the crossover period), with a total number of 629 avalanches, is also approximated rather well with a power law [see Fig. 2(b)], with an exponent $\tau = 1.15(3)$. Clearly the two exponents are not significantly different. However, a very significant difference between the two distributions is the hump in the large-avalanche regime (see arrow), due to the appearance of quasiperiodic system-spanning avalanches.

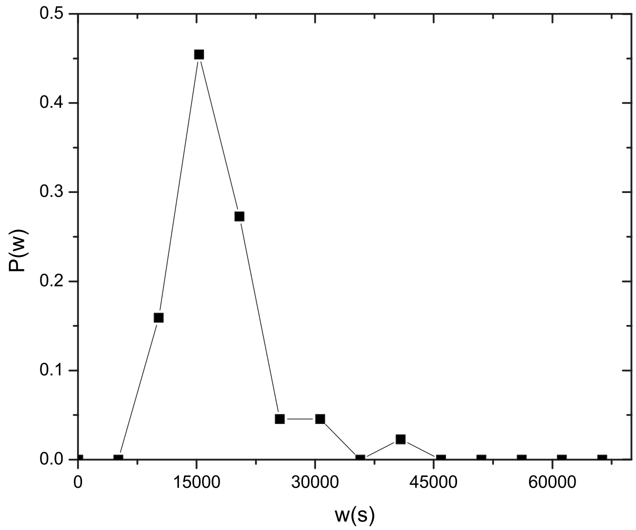

FIG. 3. The distribution $P(w)$ of waiting times w between the 44 observed large avalanches (larger than 8 dm^3) for the type-II boundary condition, showing a predominant interval between large avalanches of 15 000 s. For the type-I boundary condition there were no such large avalanches observed during all our experiments.

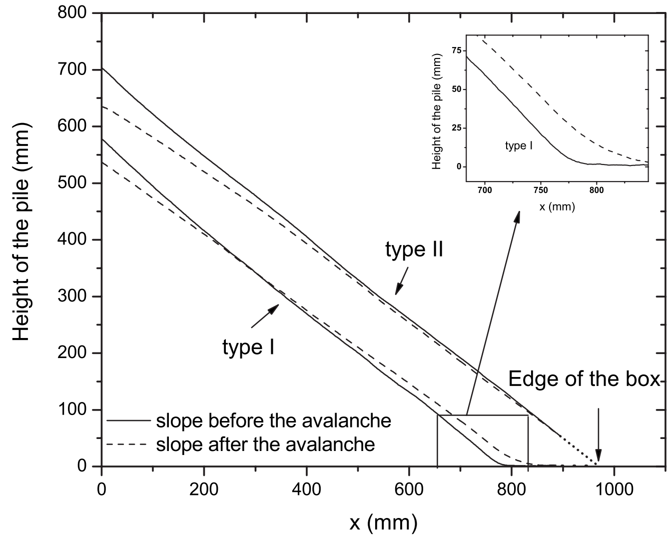

FIG. 4. Typical slopes just before and after an avalanche for the two boundary conditions, averaged over the width of the avalanche. For the type-I boundary condition, the foot of the pile rests on a horizontal surface which extends far beyond the pile, while for the type-II boundary condition, the foot is at the edge, from which grains may drop unimpeded. For the type-I boundary condition, a large avalanche causes a redistribution of grains and reduction of slope over the whole pile, while for the type-II boundary condition excess grains fall off the pile and the bottom part remains close to the critical slope. The inset shows a magnification of the foot of the pile before and after a type-I avalanche. The dotted line is an extrapolation of the slope to the edge of the box.

The quasiperiodicity of the large avalanches is revealed by the waiting time distribution between the 44 avalanches larger than 8 dm^3 (see Fig. 3). The peak at 15 000 s marks the predominant interval. This waiting time distribution is very similar to the one found between the system-spanning avalanches in the sandpile experiments (see Fig. 2. in Ref. [8]). For the avalanches with the type-I boundary condition, the corresponding distribution is $P(w) \equiv 0$: no such large avalanche sizes were observed.

The different behavior of the pile with the two different boundary conditions may be understood from the slope of the pile just before and after a large avalanche. In the case of the type-I boundary condition, the slope of the pile decreases significantly under the whole area of the avalanche (see Fig. 4; note that the slope is presented for one avalanche in each case and it is averaged over the width of the avalanche). The grains that fall down to the bottom of the pile extend the pile and thus decrease the average slope everywhere. Close to the foot of the pile (see inset in Fig. 4) the slope is even more decreased; this, of course, stabilizes the pile. However, for the type-II boundary condition the foot of the pile coincides with the edge of the box, so the grains that arrive here fall off the pile instead of decreasing its slope as in the case of the type-I boundary condition. What we observe in our experiments is that the average slope is decreased mainly at the upper part of the pile, while the bottom part remains close to the critical slope and is hence unstable: grains are on the verge of dropping from the edge. A disturbance of a single grain at the foot tends to spread immediately sideways, since its neighbors are also on the verge of dropping off the edge. As a neighboring grain falls off, it destabilizes the grain above it and so the avalanche propagates upwards, generating a broad avalanche (for a description of uphill avalanches see [19]). The combination of broadening and upwards spreading makes these avalanches particularly large (system spanning). The slope in the bottom part of the pile remains close to critical because the grains that drop from the pile are replenished by grains from higher up. By contrast, for type I, system-spanning avalanches are rare because after an avalanche the grains at the foot are rather stable, being at a much smaller slope than the rest of the pile, which is already below the critical angle (see inset of Fig. 4). Since the buildup of the slope of the upper part of the pile takes place through power-law-distributed small- and medium-sized avalanches, the type-II large avalanches are not all of equal size and do not occur periodically, but only quasiperiodically.

We conclude that the appearance of the quasiperiodic large avalanches discussed above is due to the boundary condition. If the foot of the pile is away from the edge of the box, the distribution of the avalanche sizes is a power law, while if the foot of the pile coincides with the edge of the box, quasiperiodic large avalanches can be observed, just like in the sandpile experiments. Although the earlier rice experiments [15], contained between two glass plates, had the type-II boundary condition, they did not show quasiperiodic large avalanches. This is further evidence for the idea that propagation of the avalanche along the edge (as discussed above) creates the quasiperiodic behavior: in these experiments propagation along the edge is not possible.

Power-law-distributed events in nature are unwanted because of the relatively large probability of extremely large catastrophes. Our system with the type-II boundary condition is even worse due to its preference for system-spanning (i.e., all-devastating) avalanches. By changing only the boundary condition from type II to type I, the system-spanning avalanches can be suppressed. If these results can be applied to real-world systems, the potential for catastrophe prevention is large, since just by removing ledges, the frequency of the very large system-spanning avalanches is greatly reduced.

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